Key :

Name: Student number:

Computational Science 260

Second Midterm Exam

Nov. 30, 1995

Marks

12

1. Write the predicate max(L, M) in Prolog. Here, L is a list of numbers, and max(L, M) must succeed if M is the largest element in the list. Otherwise, the predicate should fail. The predicate should also fail for the empty list.

```
max( [X], X).
max( [X | Tail ], X) :- max (Tail, Z), X > Z .
max( [X | Tail ], Z) :- max (Tail, Z), X <= Z.
```

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CH5 2. Let $A = \mathbb{P}\{3\}$. Give A is roster notation, and find $\#A$.

$A = \{ \{\}, \{3\} \}$

$\# A = 2$

$\mathbb{P}$ Powerset

$\#A$ Cardinality

$A = \{ \emptyset, \{3\} \}$ $\#A = 2$

$A = \{ \{\}, \{3\} \}$

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CH6 3. Let $f : X \nrightarrow Y$ be a partial function from X to Y. Use appropriate phrases to characterize f under the following conditions

Domain is whole
Range is a subset of Range

(a) $\operatorname{dom} f = X$: total function

(b) $\operatorname{dom} f = X, \operatorname{ran} f = Y$: surjective

(c) $\operatorname{dom} f = X, \operatorname{ran} f = Y, \underbrace{x \neq y \Rightarrow f(x) \neq f(y)}_{\text{injective}}$: bijective

ONTO

1:1

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CH5 4. Let A be a set of people who have attended party 1, and let B be the set of people who have attended party 2. Furthermore, C be the set of people that have attended both parties, and let D be the set that have one, but not both. Use the normal set opertions, such as union, intersection, etc, to express C and D in terms of A and B.

Tricky $C = A \cap B$

9 Tricky $D = (A \cup B) - (A \cap B)$

5. Two relations R and S are given as follows

CH5

$$R = \{(mary, john), (jane, brent), (lia, paul), (anne, ken)\}$$

$$S = \{(lia, carl)\}$$

Find the set $A = \{(x, y) \mid x \notin \text{dom}\, S \wedge xRy\} \cup S$ in roster notation.

$A = \{(mary, john), (jane, brent), (lia, carl), (anne, ken)\}$

10 A updates all information about lia.

CH6 6. Let S be the sibling relation, that is, $(x, y) \in S$ iff x and y have both parents in common. Let H be the halfsibling relation, that is $(x, y) \in H$ iff x and y share the father or the mother, but not both. Furthermore, let I be the identity relation.

(a) Prove that $S \cup I$ is an equivalence relation by verifying that all the properties required for an equivalence relation are met.

(b) Is $H \cup I$ and equivalence relation? Check all the properties required for a relation to be an equivalence relation, and indicate which ones are met.

a) 1. $S \cup I$ is reflexive because I is
2. $S \cup I$ is symmetric: If x is sibling of y, y is sibling of x
3. $S \cup I$ is transitive.

b) 1. $H \cup I$ is reflexive because I is
2. $H \cup I$ is symmetric
3. $H \cup I$ is <u>not</u> transitive: A halfsibling of a halfsibling is normally not a half-sibling.

① FIND Relation
② D/T Matrix
③ Calcul

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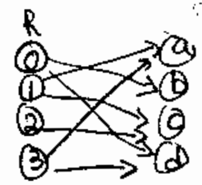
CH6

7. Let $R : 0..3 \leftrightarrow \{a, b, c, d\}$ be a relation, and let the relation matrix of R be given as follows

TRICKY

$$M_R = \begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$



~R
Look backwards
{(a,1) (a,3) (b,0) (c,1) (c,2) (d,0) (d,3)}

R∘R~ {0-b b-0 ...

Give the relation $R \circ R^{\sim}$ in roster notation.

Do

$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array} \odot \begin{array}{c|c} & \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array} = \begin{array}{c|c} & \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

m_R $\quad$ $m_{R^{\sim}}$

in roster notation:

{(0,0), (0,3), (1,1), (1,2), (1,3), (2,1), (2,2), (3,0), (3,1), (3,3)}

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8. Let *real* be a basic type of Z, and let x : *real*, y : $\mathbb{Z}$. Give the Z declaration of the function f defined by $f(x, y) = x + y$. Here, $x + y$ is evaluated as in Pascal, that is, mixed mode expressions yield a real result.

$f : real \times \mathbb{Z} \to real$

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9. Consider the following Z fragment which implements a phone directory.

[*name*, *phone*]

$message ::= ok \mid not\_in\_directory$

book

$directory : name \rightarrowtail phone$

$subscribers = \mathrm{dom}\ directory$ subscriber

(a) Write two schemas for finding the phone number of a subscriber. The first of these two schemas should apply for the case where the name, call it $x?$, is in the directory, and the second schema should deal with the case where $x?$ cannot be found in the directory.

(b) Suppose the declaration of *directory* is changed from $directory : name \twoheadrightarrow phone$ to $directory : name \leftrightarrow phone$. Write a schema for this case, in which all phone numbers of $x?$ are output.

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a)

find_number

$\Xi\ book$

$number! : phone$

$x? : name$

$confirmation! : message$

$x? \in subscriber$

$number! = directory\ x$

$confirmation! = ok$

not_listed

$\Xi\ book$

$x? : name$

$confirmation! : message$

$x? \notin subscriber$

$confirmation! = not\_in\_directory$

b)

find_numbers

$\Xi\ book$

$numbers! : \mathbb{P}\ phone$

$x? : name$

$confirmation! : message$

$numbers! = \{ y : phone \mid x? \mapsto y \in directory \}$

Also possible:

$numbers! = directory\ (\!|\ \{x?\}\ |\!)$.